A Runge-Kutta Primer

A well known technique for solving first-order differential equations is the *Runge-Kutta-Gill* method. For the equation

$$\frac{dy}{dx} = f(x, y)$$

the solution takes the form

$$y_{i+1} = y_i + \frac{\Delta x}{6}\left[k_1 + \left(2 - \sqrt{2}\right) k_2 + \left(2 + \sqrt{2}\right) k_3 + k_4\right]$$

where

$$\begin{aligned}
k_1 &= f\left(x_i, y_i\right) \\
k_2 &= f\left(x_i + \frac{\Delta x}{2}, y_i + k_1 \frac{\Delta x}{2}\right) \\
k_3 &= f\left(x_i + \frac{\Delta x}{2}, y_i + \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) \Delta x k_1 + \left(1 - \frac{1}{\sqrt{2}}\right) \Delta x k_2\right) \\
k_4 &= f\left(x_i + \Delta x, y_i - \frac{\Delta x}{\sqrt{2}} k_2 + \left(1 + \frac{1}{\sqrt{2}}\right) \Delta x k_3\right)
\end{aligned}$$

If you can compute k_1, which is the right hand side of the ODE, at the initial condition (x,y), then you can solve the problem. The ODE solver *lsode* in Octave or *ode45* in Matlab is much more sophisticated than the *Runge-Kutta-Gill* method and it efficiently selects and adapts the increment Δx to make on each iteration. However, the concept behind how each method works is the same and having an appreciation of what goes on in the solution will help you understand what is needed to properly define the problem.